Didactic Guide

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BALLISTIC PENDULUM

OPTIKA S.r.l.
Via Rigla, 30 – 24010 Ponteranica (Bergamo) – Italy
Tel. +39 035 571392 Fax +39 035 571435

www.optikascience.com info@optikascience.com

TABLE OF CONTENTS

LIST OF MATERIALS

1 PLATFORM WITH LAUNCHING SYSTEM	OFF4057
1 PHYSICAL PENDULUM	OFF4071
1 LINCHPIN	OFF4078
1 T-SHAPED FULCRUM	OFF4080
2 STEEL SPHERES (diameter 14 mm)	OFF4081
1 FOLDING RULER	1116
1 ALLEN KEY n. 4	OFF1403C
2 M5 SOCKET HEAD SCREWS	

Not supplied materials needed:
1 Stopwatch
1 Electronic scale

SUPPLIED MATERIALS

WARNING: SAFETY RULES

The launching system must be used very carefully, a careless use of this instrument could cause injury to users. For this reason, please observe the following instructions:

- Make sure no one is on the launch trajectory!
- Don’t look into launcher barrel to control the ball in inside.
- Wear protective goggles during the experiments.
- Launching system should be always stored without ball into the barrel and the spring should be unloaded.
- You must use specific extractor to check ball position.

HOW TO ASSEMBLY

The pillar that supports the compound pendulum must be fixed to base using the two supplied screws and allen key, as shown in Figure A.
Once this is done, make sure the launching system and the pendulum are aligned properly.
Doing experiences using the launching system in the position of minimum power. The ball can be removed by inserting the specific linchpin into the back hole in the pendulum body.

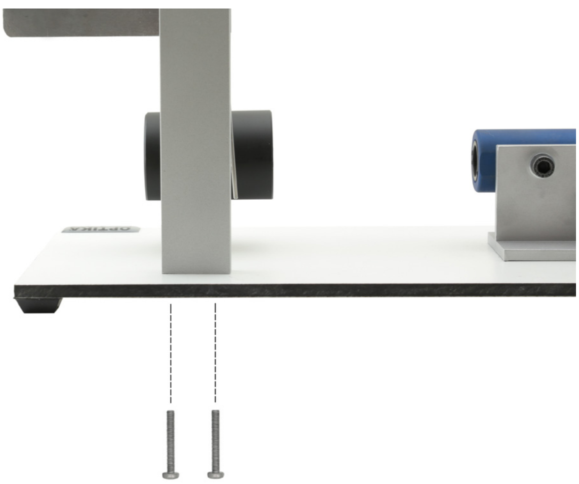

Fig. A

Fig. B

HELPFUL HINTS

Make these experiences on a stable work surface. Load the launching system to the desired position and insert the ball very carefully. Use the specific to make sure that the ball is in contact with the deepest point of cannon barrel. Adjust the pressure acting on the angle indicator support, by gently swirling the cylinder PVC indicated by the arrow in figure B. Make sure that the angle indicator does not oppose a great resistance to pendulum rotation and, on the other hand, it maintains its position once moved.

1) INTRODUCTION

The ballistic pendulum allows to evaluate the launch velocity of a projectile thanks to the law of angular momentum conservation. This apparatus consist of a launching system, used to launch a steel sphere, and a physical pendulum to measure velocity. The pendulum is composed of a PVC block suspended by an aluminium rod able to rotate around an axis. Once launched, the projectile enters the conical hole of the pendulum and remains stuck in it. Right after the collision, the two bodies move at the same velocity while the graduated scale indicates the maximum rotation of the “pendulum+projectile” system.
If the collision is inelastic and its duration is far smaller than the oscillation period of the system, the block remains still for the entire collision duration.

2) WHEN THE OSCILLATING SYSTEM IS A PHYSICAL PENDULUM - THEORY

The use of a simple pendulum to assemble a ballistic pendulum involves a series of practical difficulties which can be avoided using a physical pendulum like the one provided with this apparatus, represented in figure 1.

Fig. 1

It is convenient to divide the theoretical discourse into two stages: before and after the collision.

First stage: before the collision

Since the collision between the projectile and the physical pendulum is inelastic, kinetic energy is not conserved.
Therefore, only the law of angular momentum conservation can be applied to this phenomenon.

Angular momentum

Projectile angular momentum of the with respect to the pendulum’s centre of rotation O: $\Lambda_1 = mvL$

Pendulum angular momentum of the before the collision: $\Lambda_2 = 0$

System angular momentum of the before the collision: $\Lambda_i = \Lambda_1 + \Lambda_2 = mvL$

where *m* is the mass of the projectile, *v* its velocity and *L* is the distance between the direction *x* of the projectile’s motion and the pendulum’s centre of rotation (fig. 2).

Second stage:

When the projectile enters pendulum hole, it puts pendulum in motion. However, the initial velocity of the pendulum is not the same as the velocity the projectile had before the collision.

Angular momentum

Angular momentum of the pendulum+projectile system after the collision: $\Lambda_f = I_t \omega$

where I_t is the total moment of inertia of the system, while ω is its angular velocity (fig. 3).

According to the law of angular momentum conservation, $\Lambda_i = \Lambda_f$. That is,

$$mvL = I_t \omega \qquad (1)$$

Energy

The kinetic energy of the pendulum+projectile system immediately after the collision is rotational energy:

$$K_f = \frac{1}{2} I_t \omega^2$$

As soon as the pendulum starts oscillating, its barycentre *G* moves upwards and reaches the maximum height after a quarter of a period.
Its initial kinetic energy is transformed into gravitational potential energy *U.*
In correspondence with the maximum elongation $K_f = U$.
That is,

$$\frac{1}{2} I_t \omega^2 = (M + m)\, gh \qquad (2)$$

where *g* is gravitational acceleration, *M* is the mass of the pendulum and *h* is the displacement of the centre of mass in correspondence with the maximum elongation (fig. 4).

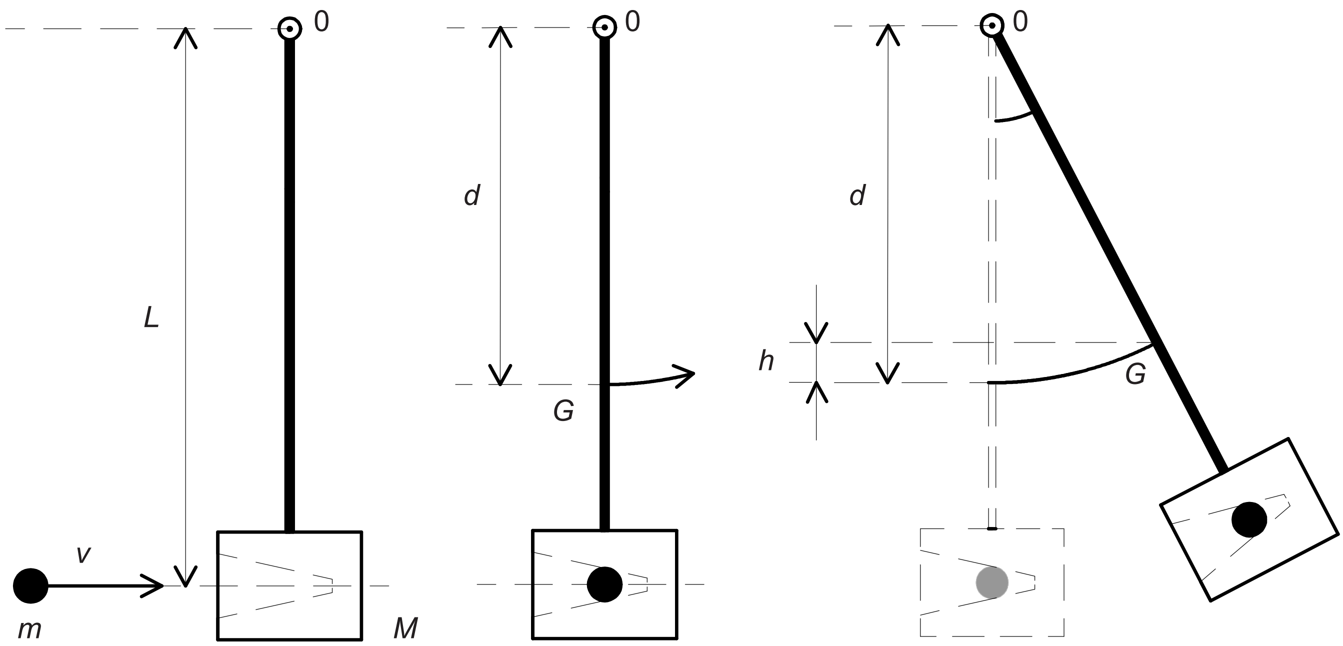

Fig. 2 Fig. 3 Fig. 4

CONCLUSIONS

Deleting ω from the comparison of relations (1) and (2) we obtain

$$\frac{m^2 v^2 L^2}{I_t} = \frac{2\,(M + m)\,gh}{I_t^2}$$

from which

$$v = \frac{1}{mL} \sqrt{2\,(M + m)\,ghI_t} \tag{3}$$

In this last relation there are two unknowns terms, h and I_t. The first term can be determined if the angle θ of maximum elongation is known (fig. 5) with the following formula:

$$h = d - d\cos\theta = d(1 - \cos\theta) \tag{4}$$

where d is the distance between the G of the physical pendulum and its centre of oscillation O.

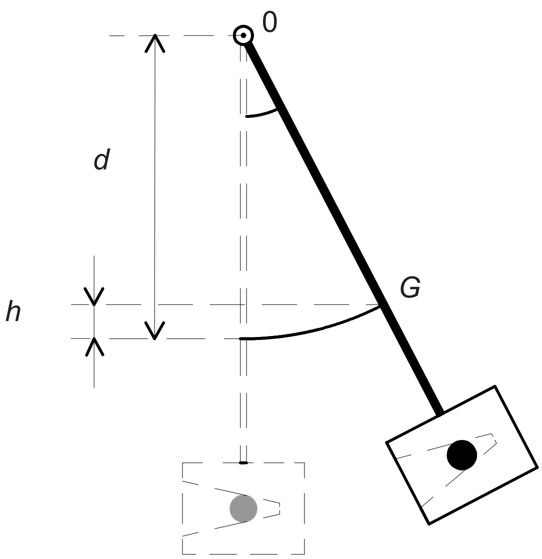

Fig. 5

The second one can be derived observing that the period of oscillation of the physical pendulum, including the sphere, is

$$T = 2\pi \sqrt{\frac{I_t}{(M + m)\,gd}}$$

hence

$$I_t = \frac{(M + m)\,g\,d\,T^2}{4\,\pi^2} \tag{5}$$

Substituting relations (4) and (5) in relation (3), we obtain

$$v = \frac{(M + m)\,gdT}{2\pi mL} \sqrt{2\,(1 - \cos\theta)} \tag{6}$$

Observation

If the amplitude of angle θ does not exceed 25°, it is possible to demonstrate that the value of the radical in formula (6), is equal, down to hundreoth, to the measure of θ in radians. That is to say,

$$\sqrt{2(1-\cos\theta)} = \theta^r$$

As a consequence, formula (6) can be rewritten as follows:

$$v = \frac{(M+m)\,gdT}{2\pi mL}\,\theta^r \qquad (7)$$

3) BALLISTIC PENDULUM EXPERIMENT

Materials needed: 1 folding ruler; 1 T-shaped fulcrum; 1 scale;1 stopwatch.

Operation 1

Unscrew the handwheel that locks the pendulum, see Figure 6. Taking care not to lose the two nylon washers, pull out the pendulum from its pivot.

Operation 2

Measure with the folding ruler, the minimum distance L between the centre of oscillation O and the x axis along which the projectile is shot (fig. 7a).

Operation 3
Using a mass balance, find the mass M of the pendulum and the mass m of the steel sphere.

Position the pendulum+projectile system on the T-shaped fulcrum in such a way that it is in equilibrium, as shown in figure 7b. Measure the distance d between the centre of oscillation O and the centre of mass G of the pendulum.

Fig.6

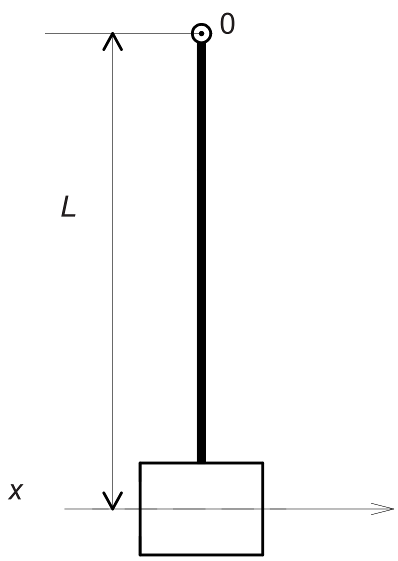

Fig. 7a

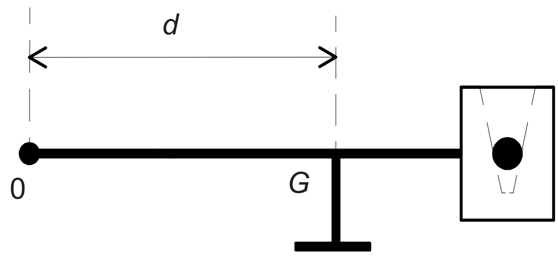

Fig. 7b

Operation 4

Insert on the support pivot axle a nylon flat washer, the pendulum and then a further nylon flat washer. Screw the handwheel as shown in figures 8a and 8b, adjusting its position so that the pendulum is free to oscillate. Insert the sphere in the horizontal hole of the pendulum and slightly push the pendulum, to make it oscillate. Determine with the stopwatch the time *t* during which the pendulum describes ten complete oscillations. The period is

$$T = \frac{t}{10}$$

Operation 5

To determine the angle θ, load the launching system's spring. Adjust PVC cylinder that keeps tension in the angle indicator: it must move with minimal friction only when it impacts with the pendulum.
After placing the angle indicator at zero, launch the ball. The index will mark the maximum elongation angle (fig. 9).

Fig. 8a

Fig. 8b

Fig. 9

Once the values of the quantities *M, m, L, d and T* are known, you can use formula (6) or (7) to evaluate the velocity v of the projectile.
You can perform this experiment with different launching velocities. For each chosen value of velocity, it is recommended to leave the angle indicator in the position it reached and to repeat the experiment again. In such a way, the second prove is not affected by the friction generated by it.

4) CHECK THE RESULT

Materials needed: 1 folding ruler; 1 white paper sheet; 1 carbon paper sheet.

Let us verify the result obtained in the previous experiment.

Operation 1

Unscrew the two handwheels that fix the launching system to the platform. Rotate180° the launching system and fix it to the platform, then put the appartus on the edge of a table, as shown in figure 10. Measure with the ruler the distance y_0 between the floor and the centre of the launching system barrel (fig. 11).

Fig. 10

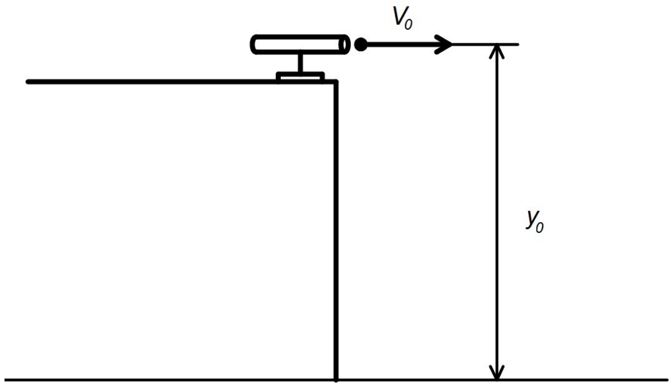

Fig. 11

Operation 2

Load the spring in correspondence with the first position, then launch the sphere.
Identify the area of the floor where the sphere falls and repeat the experiment positioning in that area a white paper sheet with a carbon paper sheet on it. So you will be able to evaluate the range x_R (fig. 12).

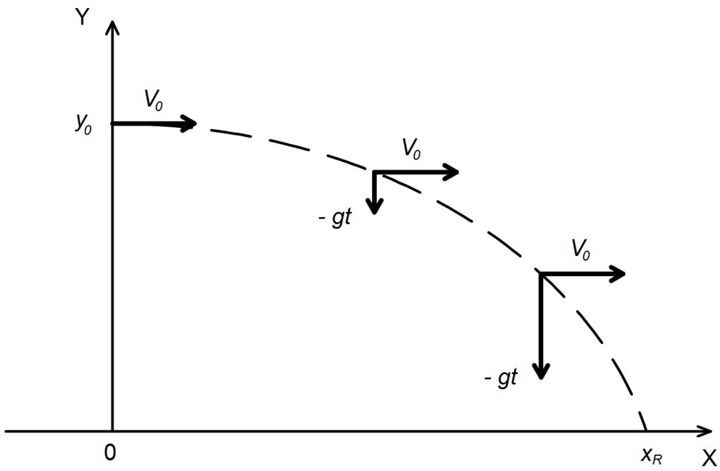

Fig. 12

Projectile's motion is the result of the superposition of two motions:

- horizontal motion, constant speed $v_x = v_0$;
- vertical motion, constant acceleration $-g$, hence velocity results to be $v_y = -g\,t$.

Motion equation along the two axes are:

$$x = v_0\, t \qquad\qquad y = y_0 - \frac{1}{2} g\, t^2$$

The time t_R that the projectile takes to reach the ground ($y = 0$) can be calculated using the second equation:

$$t_R = \sqrt{\frac{2\, y_0}{g}}$$

hence the range is:

$$x_R = v_0 \sqrt{\frac{2\,y_0}{g}}$$

and the launch velocity:

$$v_0 = x_R \sqrt{\frac{g}{2\,y_0}}$$

Repeat the experiment using each loading positions to evaluate the five different launch velocities and to compare them to those you obtained with the ballistic pendulum.

NOTE

The small differences between the characteristics of the pieces making the collection and the relative drawings are justified by technological upgrade.